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TRANSMISSION-MATRIX REPRESENTATION OF FIN-LINE DISCONTINUITIES

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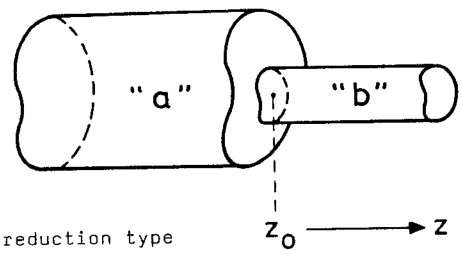
Summary

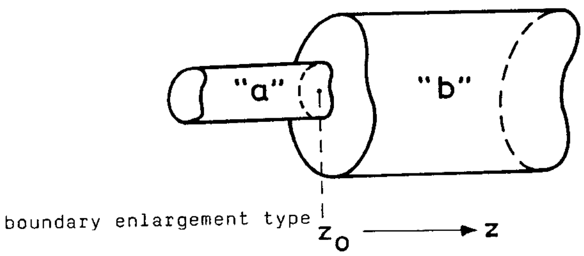
A general treatise of cascaded discontinuities is given and applied to fin-line circuits. A transmission matrix representation is superior to a scattering matrix representation as far as cpu-time is concerned. The scattering matrix is, however, advantageous if the line length between any two junctions is large. Numerical examples are given for illustration.

INTRODUCTION

Fin-line circuits usually consist of a large number of line sections separated by abrupt junctions. In the case of multi-section bandpass filters e.g., the dimensions of the slot pattern must be altered many times until the required response curve is well approximated. Hence saving cpu-time is an important factor in system design. We will compare transmission and scattering matrix representations of cascaded junctions and develop some guidelines for the design of a complex fin-line circuit. The results are, however, general and can be applied to any other circuit technology.

Most of the waveguide discontinuities have either the forms shown in Fig. 1: a boundary reduction -type /1/, a boundary enlargement-type /1/, and a mixed-type discontinuity. Following /1/ but allowing for an arbitrary number of incident modes one can





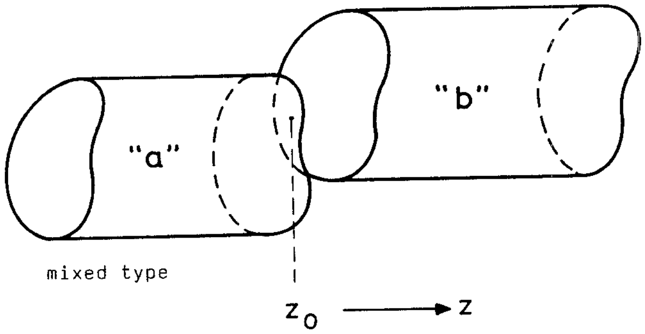


Fig. 1 Different types of waveguide discontinuities

relate the complex amplitude vectors $\underline{a}^+$ and $\underline{b}^-$ of the incident modes to $\underline{a}^-$ and $\underline{b}^+$ of the scattered modes for the case of a boundary reduction-type problem by

$$\underline{\underline{\lambda}}_P(\underline{a}^+ + \underline{a}^-) = \underline{\underline{A}}(\underline{b}^+ + \underline{b}^-), \quad \underline{\underline{\lambda}}_Q(\underline{b}^+ - \underline{b}^-) = \underline{\underline{A}}^t(\underline{a}^+ - \underline{a}^-). \qquad (1)$$

Here superscripts + or - refer to propagation in +z or -z-direction, respectively. Furthermore, the number of modes in guides "a" and "b" has been limited to N and M, respectively. Then $\underline{\underline{\lambda}}_P$ is an N•N diagonal matrix, $\underline{\underline{\lambda}}_Q$ an M•M diagonal matrix, and $\underline{\underline{A}}$ an N•M matrix whose elements can be derived by generalizing the analysis given in /1/.

Boundary enlargement discontinuities are described by relations which are similar to eqs. (1) with slightly different matrix elements. For discontinuities of the mixed type we proceed as sketched in Fig. 2: Such junction is treated as two junctions in cascade, the first "a"-"c" being of the boundary reduction, the second "c"-"b" being of the boundary enlargement type. Line length l of "c" is then set to zero. The principle of conservation of complex power (see e.g. /2/) is used for checking the accuracy of the so-obtained results.

SCATTERING AND TRANSMISSION MATRIX REPRESENTATIONS

From an inspection of eqs. (1) one recognizes that if the number of variables N≠M it is impossible to express $\underline{a}^+$, $\underline{a}^-$ in terms of $\underline{b}^+$, $\underline{b}^-$. The most suitable choice of dependent and independent variables is to express $\underline{a}^-$, $\underline{b}^+$ in terms of $\underline{a}^+$, $\underline{b}^-$. Each of these pairs represents (N+M) variables. This gives rise to the scattering matrix representation

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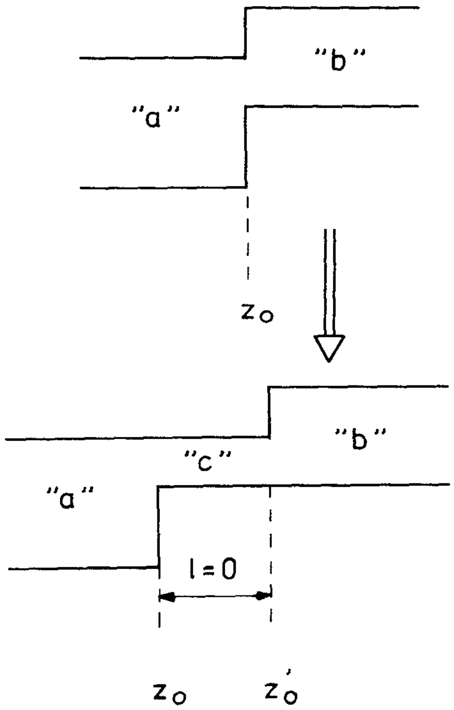

Fig. 2 Equivalence between mixed type discontinuity and two cascaded normal discontinuities

$$\begin{vmatrix} \underline{a}^- \\ \underline{b}^+ \end{vmatrix} = \begin{vmatrix} \underline{\underline{S}}_{11} & \underline{\underline{S}}_{12} \\ \underline{\underline{S}}_{21} & \underline{\underline{S}}_{22} \end{vmatrix} \cdot \begin{vmatrix} \underline{a}^+ \\ \underline{b}^- \end{vmatrix} \qquad (2)$$

where the individual sub-matrices can be calculated from $\underline{\underline{\lambda}}_P$, $\underline{\underline{\lambda}}_Q$, and $\underline{\underline{A}}$. The corresponding procedure involves at least 1 matrix inversion and 5 matrix multiplications.

The situation is much simpler for an equal number of modes in guides "a" and "b": N=M. One can then relate the corresponding modes of each waveguide by the transmission matrix according to

$$\begin{vmatrix} \underline{a}^+ \\ \underline{a}^- \end{vmatrix} = \begin{vmatrix} \underline{\underline{U}} & \underline{\underline{V}} \\ \underline{\underline{V}} & \underline{\underline{U}} \end{vmatrix} \cdot \begin{vmatrix} \underline{b}^+ \\ \underline{b}^- \end{vmatrix} \qquad (3)$$

where $\underline{\underline{U}}$ and $\underline{\underline{V}}$ are calculated from $\underline{\underline{\lambda}}_P$, $\underline{\underline{\lambda}}_Q$, $\underline{\underline{A}}$ by only 1 matrix inversion. Another advantage of this representation is the easy handling of mixed-type discontinuities, where the resultant sub-matrices $\underline{\underline{U}}$ and $\underline{\underline{V}}$ are calculated from multiplying the individual sub-matrices.

In the case of cascaded discontinuities there are 2 approaches: The first is to process the individual scattering matrices (see e.g. /3/). This requires at least 1 matrix inversion and 10 matrix multiplications if two discontinuities are to be treated. The second is to process the individual transmission matrices for N=M. This requires 8 matrix multiplications and no inversion (see Fig. 3).

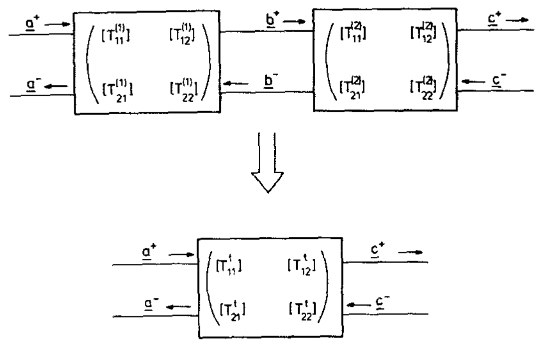

Fig. 3 Transmission matrix representation of cascaded discontinuities

For discontinuities between waveguides having the same housing as e.g. fin-lines and microstrip lines, it has already been shown /3/, /4/ that there is no need to take different numbers of modes in the individual waveguides into account. Hence the transmission matrix representation for cascaded discontinuities is by far the fastest one as far as computer time is concerned. Due to the finite thickness of the metal fins, three different junctions are imaginable (Fig. 4): a decreasing (increasing) slot width corresponds to the boundary reduction (enlargement) case, while a shifted slot axis corresponds to the mixed-type problem. All these discontinuities should preferably be described by the transmission matrix.

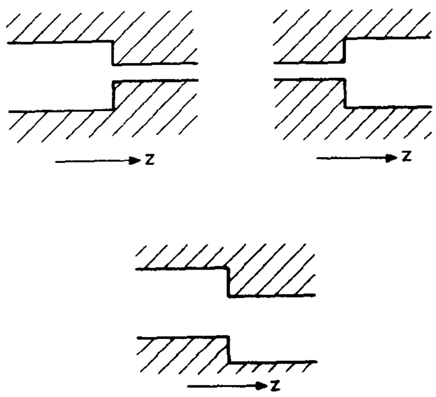

Fig. 4 Different fin-line discontinuities

THE OVERFLOW PROBLEM

Cascaded discontinuities are always separated by uniform line sections, which may give rise to computational problems. The elements of their scattering matrix are composed of terms $z_i = \exp(-\gamma_i l)$, while the transmission matrix contains z_i as well as $1/z_i$. For evanescent modes, the latter is usually a very large number which may lead to an overflow in particular if multiplications have to be performed. In case of the transmission matrix representation overflow might occur if the sum of all separating line sections exceeds the attenuation distance of the highest-order mode. (This distance is defined as $1/\gamma_i$, γ_i being the corresponding attenuation constant.) In this case one should collect the cascaded discontinuities in groups which are separated by line sections which are long in the above sense, while the line sections within each group are small. Each group is then treated by the transmission matrix representation, while the groups are cascaded within the scheme of scattering matrices.

NUMERICAL RESULTS

The validity of our approach of analyzing mixed-type fin-line discontinuities has been checked by computing the complex power just before and just behind the junction. The fields on either side have been expanded into 5, 10, or 15 modes. The results in Table 1 show, that the complex power is conserved irrespective of the number of modes.

The cpu-time of both a transmission and a scattering matrix representation of cascaded discontinuities is shown in Table 2. From this point of view, the former method is preferable. - The influence of the number of modes is finally illustrated in Fig. 5 showing s_{11} versus frequency of two junctions in cascade. From this a 10-mode expansion proves to be sufficient.

number of modes	complex power just before the junction	just behind the junction
5	0.9935+j0.0023	0.9935+j0.0023
10	0.9935+j0.0021	0.9935+j0.0021
15	0.9935+j0.0025	0.9935+j0.0025

Table 1: Complex power (in arbitrary units just before and just behind a mixed-type fin-line junction

number of junctions	cpu-time in seconds T-matrix formulation	S-matrix formulation
1	30	46
2	65	106
3	109	171
4	150	246
5	183	310

Table 2: Comparison between cpu-time needed for T- and S-matrix formulations

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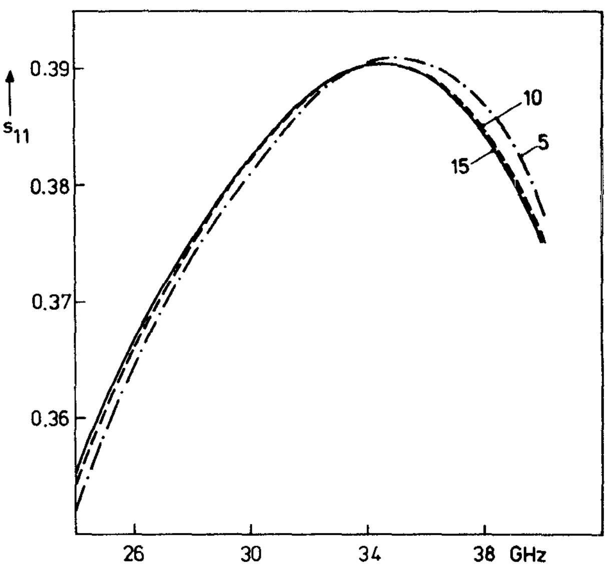

Fig. 5 Effect of number of modes on the frequency response of S_{11}